

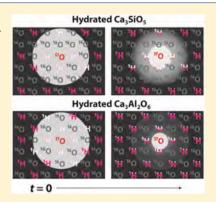
Measurement of Proton Spin Diffusivity in Hydrated Cementitious **Solids**

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Supporting Information

ABSTRACT: The study of hydration and crystallization processes involving inorganic oxides is often complicated by poor long-range order and the formation of heterogeneous domains or surface layers. In solid-state NMR, ¹H-¹H spin diffusion analyses can provide information on spatial composition distributions, domain sizes, or miscibility in both ordered and disordered solids. Such analyses have been implemented in organic solids but crucially rely on separate measurements of the ¹H spin diffusion coefficients in closely related systems. We demonstrate that an experimental NMR method, in which "holes" of well-defined dimensions are created in proton magnetization, can be applied to determine spin diffusion coefficients in cementitious solids hydrated with ¹⁷O-enriched water. We determine proton spin diffusion coefficients of 240 \pm 40 nm²/s for hydrated tricalcium aluminate and 140 \pm $20 \text{ nm}^2/\text{s}$ for hydrated tricalcium silicate under quasistatic conditions.



 ${f S}$ olid-state NMR can often resolve different components of a mixture and is a method of choice for characterizing complex domains on the nanoscale.^{1,2} Conventional examples include characterization of domain sizes in heterogeneous and semicrystalline polymers and spatial composition distributions in lipid membranes.³⁻⁵ Recent advances involving the relay of nuclear hyperpolarization through interfaces,^{6,7} as generated by dynamic nuclear polarization,^{8–10} are proving particularly effective in this regard. In such experiments, proton hyperpolarization in one phase sets up large magnetization gradients at interfaces with a second phase of interest. As hyperpolarization builds up outside the second phase, efficient proton spin diffusion spontaneously and simultaneously transports magnetization into it. Magnetization gradients large enough to permit domain size analysis can also be established by domain selective enhancement of relaxation by doping with paramagnetic species¹¹ or by selective saturation.^{3,}

Such domain size analyses rely on knowledge of proton spin diffusion coefficients, $D_{\rm H}$, which are usually estimated based upon proton density or chemical similarity in the rigid limit. In more complex systems (e.g., calcium silicate or aluminate hydrates), structural disorder, chemical heterogeneity, or molecular mobility of intralayer water may influence $D_{\rm H}$ values. An experimental determination of proton spin diffusion coefficients usually depends on prior knowledge of the length scale of initial magnetization gradients, L, as the NMR observables are functions not of $D_{\rm H}$ alone but rather the characteristic diffusion time $L^2/D_{\rm H}$. In many cases, L is governed by the sizes of the domains being studied and is thereby connected to the unknown parameters of prior

interest. In most systems, domain sizes cannot be reliably controlled, which prevents direct measurement of spin diffusion coefficients.

Here we show this paradox can be overcome for silicates and aluminates with an NMR experiment that burns "holes" of well-defined size in proton magnetization,^{15,19} thus establishing L regardless of any particular chemical character of the hydrated phase. The experiment we develop here is based upon the method introduced by Chen and Schmidt-Rohr,¹⁵ which utilizes spin-pair dephasing in order to burn approximately spherical holes in the proton magnetization. The holes can exceed 1 nm in diameter, and subsequent hole filling by spin diffusion could be monitored through the same nucleus (in their case, ¹³C) used to burn the hole, provided a proton was bonded to it. In adapting their approach to cementitious solids, specifically tricalcium aluminate (Ca₃Al₂O₆) and tricalcium silicate (Ca₃SiO₅) after hydration, we chose to use ¹⁷O nuclei (as shown in Figure 1) for hole burning, because of the ease of incorporation using ¹⁷Oenriched water and because of the direct covalent bonds between the hydrogen and oxygen atoms. These structural characteristics are illustrated in Figure 1.

Our implementation of the hole-burning pulse sequence under slow magic-angle spinning (MAS) is shown in Figure 2. Following saturation of ¹⁷O magnetization, the magnetic dipoles of ¹⁷O nuclei are used to destroy nearby ¹H

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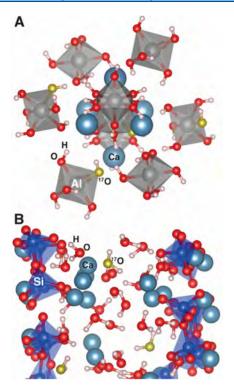


Figure 1. Schematic structures of hydrated tricalcium aluminate (A) and hydrated tricalcium silicate (B). By using water isotopically enriched in 17 O (golden atoms), spin labels are introduced that permit controllable burning of holes in the 1 H magnetization density.

magnetization. The application of a homonuclear proton decoupling method during dephasing, such as the BR-24 technique used here,²² is essential in two ways. First, it arrests proton spin diffusion. Second, it allows the dephasing dynamics to be approximated by spin-pair processes. Under

these two conditions, a hole is burned in the proton magnetization. Chen and Schmidt-Rohr defined the radius of the hole to be the distance from the heteronucleus at which the ¹H magnetization rises to half the value it reaches well outside of the hole, ¹⁵

$$r_{0.5} = \sqrt[3]{\frac{5}{4}Q\tau_{\rm b}} \tag{1}$$

Here $\tau_{\rm b}$ is the hole-burning interval, and Q is a dephasing constant

$$Q = \lambda_{cs} h \frac{\mu_0}{4\pi} \frac{|\gamma_1 \gamma_S|}{4\pi^2}$$
⁽²⁾

which depends on the scaling factor λ_{cs} accounting for the attenuation of the ¹H–¹⁷O heteronuclear dipolar interaction by BR-24, the gyromagnetic ratios γ_i of the *I* and *S* spins, and the fundamental constants μ_0 and *h*. The *S* spin here, ¹⁷O, is quadrupolar (S = 5/2), but because central transition selective π pulses were used (labeled π_{CT} in Figure 2), eq 2 remains valid.²³ Taking $\lambda_{cs} = 0.38$ for BR-24,²⁴ we calculate Q = 6.2 Å³/ms for our implementation of pairwise ¹H–¹⁷O dipolar dephasing.

Following the hole-burning interval, the proton magnetization refills the hole by spontaneous proton spin diffusion. Under the experimental conditions used, transport of proton magnetization is, to a good approximation, governed by the isotropic diffusion equation

$$\frac{\partial M}{\partial t} = D_{\rm H} \nabla^2 M \tag{3}$$

where *M* is a scalar field representing the longitudinal proton magnetization density, and $D_{\rm H}$ is the isotropic proton spin diffusion coefficient. If the ¹⁷O spin labeling is sparse, we can solve eq 3 assuming the holes are far enough apart to be

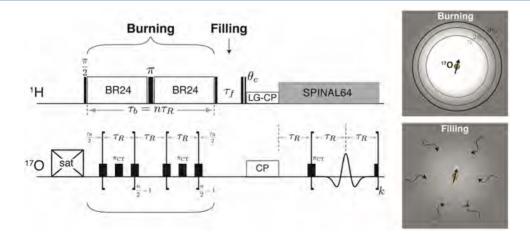


Figure 2. Hole-burning pulse sequence used for measuring proton spin diffusion coefficients in cementitious solids, based on the pulse sequence introduced by Chen and Schmidt-Rohr.¹⁵ During the hole-burning interval, REDOR¹⁶ pulses that are selective for the ¹⁷O central transition (π_{CT}) are used to dephase the polarization of nearby protons (¹H). This is represented by a "hole", the radius of which grows in proportion to $\tau_b^{1/3}$. This is depicted in the tile labeled "Burning", where the proton magnetization is depicted as a gray continuum. During this step, homonuclear ¹H decoupling is applied to suppress proton spin diffusion to validate the use of spin-pair dephasing principles for calculating the spatial profile of the hole. During the hole-filling interval, magnetization returns to the ¹⁷O nucleus by spontaneous proton spin diffusion, as illustrated in the tile labeled "Filling". Following this, the ¹H magnetization near the center of the hole is monitored by magnetization transfer to ¹⁷O using a short (10 μ s) CP contact pulse with LG irradiation¹⁷ of ¹H rf to again suppress proton spin diffusion during the transfer. Multiple-echo acquisition with CPMG¹⁸ is used to enhance sensitivity. The signal intensity is analyzed as a function of τ_f for different τ_b to determine the proton spin diffusion coefficient D_H using an analytic diffusion model. In all of our experiments, the sample rotation period for magic-angle spinning, τ_{R} was 0.33 ms. Additional experimental details are given in the Supporting Information.

treated as an ensemble of isolated, spherically symmetric wells, which yields M(r, t) as a function of the radial coordinate and time. The solution for the recovery of magnetization at the center of a spherical well, defined by the boundary conditions

$$M(r, 0) = \begin{cases} M_1 \text{ if } r < R \\ M_2 \text{ if } r > R \end{cases}$$
$$\frac{\partial M}{\partial r} \bigg|_{r=0} = 0, \forall t$$
$$\lim_{r \to \infty} M(r, t) = M_2, \forall t$$

is given by

$$M(0, t) = M_2 + (M_1 - M_2) \left[erf\left(\frac{R}{\sqrt{4D_H t}}\right) - \frac{R}{\sqrt{\pi D_H t}} exp\left(-\frac{R^2}{4D_H t}\right) \right]$$
(4)

The spherical well profile is defined by an abrupt change of magnetization from M_1 within the burned hole to the bulk polarization value M_2 at the edge of the hole at r = R. Although such an initial profile of magnetization is unrealistic, the solution can be adapted for an initial profile that features a smoother transition across the edge by introducing the advancement parameter, Δt , and making the substitution $t \rightarrow t + \Delta t$ in eq 4. In this way, the initial profile is modeled as the solution of diffusion into a spherical well at a nonzero time $t = \Delta t$, and eq 3 does not need to be solved for a more complicated set of boundary conditions.¹⁵

As discussed, eq 3 does not possess a term corresponding to a magnetization sink. In reality, our data is affected by longitudinal ¹H spin relaxation, characterized by the time constant, T_1 , which is on the order of milliseconds. The effects of relaxation, however, become significant only after most of the magnetization has returned to the hole by diffusion (vide infra), such that the behavior of magnetization exists approximately in two separate regimes, one diffusiondominated and one relaxation-dominated. For the latter case, the magnetization behaves like $M(t) = M_0 \exp(-t/T_1)$ for some initial value of magnetization M_0 . Given this approximate separability, we take M_0 to be equal to eq 4, in spite of its time dependence.

Putting all of this together, we obtain the following analytic expression to model the magnetization recovery data

$$M_{c}(\tau_{\rm f}) = \left[M_{2} + (M_{1} - M_{2}) \left[\exp\left(\frac{R}{\sqrt{4D_{\rm H}(\tau_{\rm f} + \Delta t)}}\right) - \frac{R}{\sqrt{\pi D_{\rm H}(\tau_{\rm f} + \Delta t)}} \exp\left(-\frac{R^{2}}{4D_{\rm H}(\tau_{\rm f} + \Delta t)}\right) \right] \right] \exp\left(-\frac{\tau_{\rm f}}{T_{1}}\right)$$
(5)

where the signal dependence on the parameter $\tau_{\rm f}$ has now been made explicit. We see from this expression that hole-filling data can be analyzed for M_1 , M_2 , Δt , T_1 , and the characteristic diffusion time $R^2/D_{\rm H}$. For our calculations, we take $R = r_{0.5}$ according to eq 1 to derive $D_{\rm H}$. This expresses a $\tau_{\rm b}^{1/3}$ dependence on the duration of hole burning, providing a straightforward means of controlling the size of the hole.

We note that this model of diffusion into a spherical well does not explicitly incorporate spin exchange or motional dynamics. Because these are modulated by settings such as the MAS rate and sample temperature, the set of model parameters we determine, including $D_{H\nu}$ pertains to a specific set of experimental conditions. Our experiments have been carried out at a temperature around 95 K, where significant molecular motion is frozen, and a magic-angle spinning rate of 3030 Hz, which is much lower than the strongest ${}^{1}\text{H}{-}{}^{1}\text{H}$ dipolar coupling frequencies. This prevents the suppression of spin diffusion by molecular motion and places us in a quasistatic regime where ${}^{1}\text{H}$ spin diffusion is not significantly affected (and may even be slightly enhanced) by sample rotation.^{20,21}

To selectively monitor the magnetization near the center of the hole and validate the use of eq 5, we used a very short (10 μ s) cross-polarization (CP) contact time, with off-resonance ¹H irradiation in the manner of Lee and Goldburg¹⁷ for additional suppression of proton spin diffusion during the CP transfer. Such a short contact time transfers polarization between only the most strongly dipole-coupled nuclei, which are those ¹H nuclei within a few bond lengths of the ¹⁷O nucleus at the center of each spherical well.

The magnetization recovery data and the best fit to $M_c(\tau_f)$ given by eq 5 for Ca₃Al₂O₆ hydrated with 20% ¹⁷O-enriched water for 24 h are plotted in Figure 3A. The parameters M_{2} ,

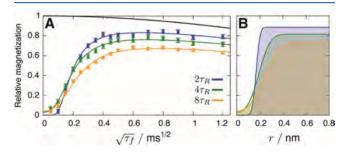


Figure 3. Profiles of ¹H magnetization for the hole-burning experiments for 20% ¹⁷O-enriched tricalcium aluminate. (A) Recovery of the ¹⁷O-detected ¹H signal intensities plotted as functions of the square root of the hole-filling interval τ_f for three values of τ_b : 0.66 ms ($2\tau_{\rm R}$; two rotor periods at the 3030 Hz MAS rate), 1.32 ms ($4\tau_{\rm R}$), and 2.64 ms ($8\tau_{\rm R}$). The curves are best fits to the experimental data (points), according to the hole-filling model, eq 5. The intensities are relative to a reference experiment without ¹⁷O REDOR pulses, in which case homonuclear ¹H decoupling is still applied but no hole is burned. The black line corresponds to the recovery limit based upon a component of spin relaxation with time constant $T_1 = 11.7$ ms. (B) Calculated radial magnetization profiles immediately after hole burning ($\tau_f = 0$), corresponding to M(r, 0), using parameters from the best-fit analysis. The sample temperature was near 95 K.

 Δt , T_1 , and D_H in eq 5 were determined as a part of the numerical fit. More details regarding the samples (including ¹⁷O enrichment levels after hydration) and fitting procedure are given in the Supporting Information (SI). The SI also shows analogues of Figure 3A for the Ca₃Al₂O₆ sample hydrated with 3% ¹⁷O-enriched water for 24 h and a Ca₃SiO₅ sample hydrated with 5% ¹⁷O-enriched water for 50 days. Our analysis of Ca₃SiO₅ is complicated by the fact that it is known to form mixtures of poorly ordered calcium silicate hydrate (C-S-H) as well as crystalline $Ca(OH)_2$ upon hydration. In the SI, we show high-resolution ¹⁷O spectra of our hydrated Ca_3SiO_5 sample (as well as the $Ca_3Al_2O_6$ samples) and carry out a quantitative analysis, which reveals that less than 20% of the 17 O is present as Ca(OH)₂. Given the poor signal-to-noise ratios of our Ca₃SiO₅ data sets, our analysis should not be led into serious error by assuming our results pertain solely to the major hydration product, C-S-H.

We see that eq 5 leads to excellent fits of three data sets acquired for hole-burning intervals of 0.66, 1.32, and 2.64 ms, corresponding to $r_{0.5}$ values of 0.17, 0.22, and 0.27 nm from eq 1. Signal intensities near zero were recorded for filling intervals less than 10 μ s, validating the constraint M_1 = 0. For $\tau_f > 10 \mu$ s, there is an onset of rapid recovery of the magnetization toward M_{2} , which represents the volume average proton polarization after hole burning. Measured with respect to a reference experiment in which the ¹⁷O dephasing pulses are deactivated and no holes are burned in the proton polarization, we should expect $M_2 < 1$, with the value of unity obtained only in the limit of infinite ¹⁷O dilution. This recovery limit is indicated by the black line in Figure 3A, and indeed, we see that in the case of hydrated Ca₃Al₂O₆, the intensities do not reach this limit. Rather, we find decreasing M_2 values of 0.88, 0.81, and 0.73 for increasing durations of hole burning. For otherwise identically hydrated Ca₃Al₂O₆ (3%), we find respective M_2 values of 1.00, 0.94, and 0.91. This trend versus $\tau_{\rm b}$ corresponds to (-0.075 \pm (0.011)/ms for Ca₃Al₂O₆ (20%) and (-0.045 ± 0.016)/ms for $Ca_3Al_2O_6$ (3%). The ratio of these values (20% over 3%) is only 1.7, yet the ratio of ¹⁷O concentrations is 6.7. This is consistent with incipient hole overlap for the $Ca_3Al_2O_6$ (20%) sample, even at the shortest hole-burning intervals used here. Further discussion pertaining to hole overlap leading to less than expected destruction of average proton polarization is given in the SI. For the Ca₃SiO₅ sample, which has a lower ¹⁷O concentration than Ca₃Al₂O₆, it is difficult to assess whether M_2 departs from unity because of low signal-to-noise ratios, and so here, we fit with the constraint $M_2 = 1$, as described in the SI.

From the full solution of eq 3, we can also calculate initial magnetization profiles M(r, 0) in the context of the analytic model for each value of $\tau_{\rm b}$, which is shown in Figure 3B using parameters determined from the best-fit analysis. The appearance of nonzero magnetization at r = 0 may appear in contradiction to the constraint $M_1 = 0$, but in fact, this is a consequence of the advancement parameter increasing from 2 to 13 to 22 μ s with increasing $\tau_{\rm b}$. This accounts for the decreasing slope of the hole edge and reflects the fact that we are not truly sampling the ¹H magnetization at the single point r = 0 but rather over a small neighborhood near r = 0, over which the integrated signal intensity tends to a small but nonzero value for sufficiently large (but finite) holes, even for ideal dipolar dephasing. Residual spin diffusion during hole burning may also contribute.

The behaviors of the M_2 and Δt parameters are consistent with expectations, increasing our confidence that $D_{\rm H}$ should behave similarly. The proton spin diffusion coefficients we determine by our analysis for each of the samples are given in Table 1. The proton concentrations, *c*, of hydrated Ca₃SiO₅ and hydrated Ca₃Al₂O₆, are around 20 and 90 M, respectively.

Table 1. Proton Spin Diffusion Coefficients, $D_{\rm H}/{\rm nm}^{2} {\rm s}^{-1}$, for Hydrated Ca₃Al₂O₆ and Hydrated Ca₃SiO₅ with Different Extents of ¹⁷O Enrichments, Determined for Different Hole-Burning Intervals

	hole-burning interval				
sample	$2\tau_{\rm R}$	$4\tau_{ m R}$	$8 au_{ m R}$	∞	
Ca ₃ Al ₂ O ₆ (20%)	156 ± 11	204 ± 15	222 ± 12	245 ± 4	
$Ca_{3}Al_{2}O_{6}$ (3%)	165 ± 11	200 ± 13	214 ± 11	232 ± 2	
Ca ₃ SiO ₅ (5%)	71 ± 7	105 ± 12	120 ± 12	137 ± 2	

Assuming that $D_{\rm H} \propto c^{1/3}$,^{25,26} we would expect $D_{\rm H}$ for hydrated Ca₃SiO₅ to be about 60% of that for hydrated Ca₃Al₂O₆, which is consistent with the values shown in Table 1.

Rather than exhibiting behaviors that are independent of $\tau_{\rm b}$ we observe in Table 1 that the spin diffusion coefficients $D_{\rm H}$ increase with the duration of hole burning $\tau_{\rm b}$. This effect is consistent across the different samples. This phenomenon was also observed by Chen and Schmidt-Rohr in their ¹³C holeburning experiments on polymer samples,¹⁵ systems for which the results of the hole-burning method could be compared with techniques that were suitable over somewhat larger length scales (roughly 10 nm). They attributed such dependence on $\tau_{\rm b}$ to anisotropic spin diffusion on nanometer length scales and to a lesser extent on nondiffusive spin dynamics on microsecond time scales. The hole-burning method was found to underestimate D_H compared to methods probing diffusion on larger length scales by a factor of about 2 or 3, depending on the size of the hole. Given the tendencies for hydrated calcium aluminates and silicate to form clustered and layered structures, similar anisotropic effects may also be factors here.

Ultimately, transport of a continuous proton magnetization by spin diffusion is a simple way to parametrize the very complicated spin exchange dynamics of protons in solids. By the parametrization of eq 3, the exchange dynamics during hole filling are such that diffusion genuinely appears to slow down as the sub-nanometer length scale is reached. Conversely, the difference between the diffusion coefficients we determine and the "macroscopic" proton spin diffusion coefficient, which we refer to as $D_{\rm H}^{\infty}$, should diminish as the size of the hole increases. By plotting our $D_{\rm H}$ values against $\tau_{\rm b}^{-1}$, a linear relationship emerges for all of our samples, as shown in Figure 4. The intercept of the trendline corresponds to the bulk spin diffusivity $D_{\rm H}^{\infty}$, which is given the rightmost column of Table 1 for the hydrated Ca₃Al₂O₆ and Ca₃SiO₅ materials with different ¹⁷O enrichments.

We find that $D_{\rm H}^{\infty}$ is about twice the value of $D_{\rm H}$ analyzed at the shortest hole-burning interval of 0.66 ms $(2\tau_{\rm R})$ for hydrated Ca₃SiO₅ and about 50% larger for the hydrated

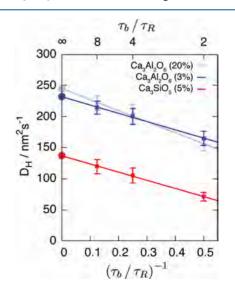


Figure 4. Plots of macroscopic proton spin diffusion coefficients $D_{\rm H}$ versus hole-burning intervals $\tau_{\rm b}$ for hydrated Ca₃Al₂O₆ and hydrated Ca₃SiO₅ with different extents of ¹⁷O enrichments obtained by extrapolation from finite hole-burning intervals.

Ca₃Al₂O₆ materials. The values and trendlines for both the 3% and 20% ¹⁷O-enriched Ca₃Al₂O₆ samples cluster together; despite clear indications of hole merging at the 20% enrichment level (vide supra), they have not yet reached levels where the diffusion coefficients we analyze are significantly affected. It would seem that our model, eq 5, compensates for the effect of hole merging primarily through the M_2 and Δt parameters. Further research would indicate the concentration of hole-burning nuclei required to incur a significant error in apparent $D_{\rm H}$.

The most significant source of uncertainty is a systematic one arising from the selection of R, which was defined by reasonable though somewhat arbitrary criteria. Instead of eq 1, the radius of natural dimensionality

$$r_{\rm nat} = \sqrt[3]{Q\tau_{\rm b}} \tag{6}$$

could have been used. If we take $R = r_{nat}$ instead of $r_{0.5}$, then the diffusion coefficients we determine will be $(r_{nat}/r_{0.5})^2 = 86.2\%$ of those reported in Table 1. On this basis, we are confident in our values of D_H to a level of $\pm 15\%$. A better procedure may be to select fixed values of R and Δt by regression to the theoretical dephasing profile. We anticipate that this uncertainty could be reduced by investigating the performance of the pulse sequence under different homonuclear decoupling schemes (to change the scaling factor λ_{cs}) and faster sample rotation (to modulate spin diffusion).^{27,28}

In summary, under quasistatic conditions such as the 3030 Hz magic-angle spinning used here, we determine the macroscopic proton spin diffusion coefficient should fall within $240 \pm 40 \text{ nm}^2/\text{s}$ for hydrated tricalcium aluminate and 140 \pm 20 nm²/s for hydrated tricalcium silicate. Physically, this means that proton magnetization is transported across space faster in hydrated tricalcium aluminate than in hydrated tricalcium silicate. We determined these values by hydrating with ¹⁷O-enriched water, which introduces spin labels that can be used to burn holes in the ¹H magnetization. The shape of the hole is well-described by dipolar dephasing principles and establishes approximate boundary conditions for the return of magnetization into the hole by ¹H spin diffusion. The isotropic radial diffusion equation yields an analytical solution to these boundary conditions, the validity of which is verified by our experimental measurements. The solution to the radial equation leads to an analytic expression for the recovery of magnetization in the hole, which fits excellently to the experimental data. The model parameters determined by our analyses can be physically rationalized, including the positive correlation of the diffusion coefficient with the duration of hole burning.

These are the first measurements of spin diffusivity in aluminates and silicates. Independent measurement of spin diffusivities allows the size of nanoscale domains to be determined with phase selectivity by, for example, magnetization relayed NMR methods.^{6,11} For silicates, this would permit tracking the appearance, size, and depth of the various hydrated phases that gradually form, crystallize, or react across the induction, acceleration, and deceleration stages of tricalcium silicate hydration.²⁹ Our results thus provide crucial parameters and methodology for future work investigating such hydration processes.

ASSOCIATED CONTENT

S Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.jp-clett.9b01861.

Sample details, additional NMR experimental and processing parameters, analysis of high-resolution 17 O spectra, reproducibility of results, fitting details, additional discussion regarding hole overlap, and the mathematics connecting eq 3 to eq 4 (PDF)

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Notes

The authors declare no competing financial interest.

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Measurement of Proton Spin Diffusivity in Hydrated Cementitious Solids

Supplementary Information

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I. Sample information

Anhydrous tricalcium aluminate (Ca₃Al₂O₆, Mineral Research Processing, France) was hydrated with either 20.9% ¹⁷O-enriched water or 3.14% ¹⁷O-enriched water for 24 h at 90 °C, with a water-to-solids ratio of 1.00 ± 0.05 . Anhydrous monoclinic tricalcium silicate (Ca₃SiO₅, Mineral Research Processing, France) was hydrated with 5.23% ¹⁷O-enriched water for 50 d at 25 °C, with a water-to-solids ratio of 0.50 ± 0.05 . The batches of 3.14% and 5.23% ¹⁷O-enriched water were made by dilution of the 20.9% ¹⁷O-enriched water with ordinary distilled water in a polyethylene vial; actual ¹⁷O concentration was determined by mass measurements.

Hydration was initiated by vortex mixing ~200 mg of the dry solid the vial containing partially ¹⁷O-enriched water for 2 min and sealing the mixture under ambient atmosphere. After hydration, the samples were submerged in liquid N₂ and lyophilized for 24 h at 0.10 Torr and -40 °C to quench the hydration process and remove unreacted bulk or weakly adsorbed water. This workup method is not expected to influence the structure of the hydrated solids in any significant way.¹ Upon hydration, Ca₃SiO₅, is known to form mixtures of poorly-ordered calcium silicate hydrates and Ca(OH)₂.² Hydration of Ca₃Al₂O₆ in the absence of sulfates results in the formation of crystalline katoite, Ca₃Al₂(OH)₁₂, which is comprised of Al(VI) species as indicated by its ²⁷Al shift shown in Fig. S1B. The density of ¹H nuclei estimated from bulk densities and stoichiometry is generally between 15 M and 30 M for fully hydrated silicates and 88 M for katoite.³

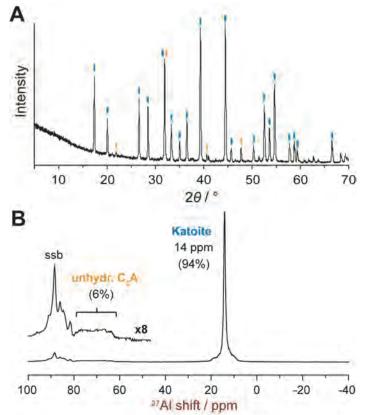


Figure S1. (A) Powder X-ray diffraction patterns with reflections indexed to Ca₃Al₂O₆ (orange) and Ca₃Al₂(OH)₁₂ respectively. (B) Quantitative ²⁷Al solid-state NMR (18.8 T, 16 kHz MAS, 25 °C) of hydrated tricalcium aluminate indicating ~94% conversion to Ca₃Al₂(OH)₁₂.

II. NMR parameters

Hold burning experiments were performed on a Bruker 900 US² wide-bore Avance Neo NMR spectrometer operating at 21.141 T, equipped with an HXY 3.2 mm DNP probe operating in ¹H/¹⁷O double mode. Samples were restricted to the central third of a rotor with inner diameter of 2.2 mm in order to maximize rf homogeneity.

To measure spin diffusion coefficients the sequence shown in Figure 2 (main text) was used. All experiments were run at a rotational frequency of 3030 Hz. BR24 was coded into a decoupling program (CPD) to ensure synchronization with the REDOR block, such that the cycle time of the decoupling was calculated to fit an integer number of full cycles inside the window permitted by the REDOR cycles. During BR24, the ¹H rf amplitude was 156.25 kHz (90° pulse length of 1.6 μ s, transmitter power near 300 W) and the length of the decoupling cycle (τ_c) was near 82 μ s. The spin diffusion coefficients were measured using REDOR dephasing periods of 2, 4, 8 and $16\tau_{\rm r}$ (corresponding to 0.66, 1.32, 2.64 and 5.28 ms), and hole filling interval τ_f up to 1.44 ms. For each REDOR dephasing time, sixteen reference spectra with τ_f of 1.44 ms (repeated to increase certainty of the recovery limit intensity) and without REDOR dephasing were acquired and used to normalize the intensities to a scale of relative magnetization. The transfer of the magnetization from ¹H to ¹⁷O was done using LG-CP with a contact time of 10 µs in order to minimize spin diffusion and ensure the transfer is local, permitting the approximation that the signal observed corresponds to the center of the hole. Immediately prior to this, the ¹H magnetization was placed along the effective field with a 35° pulse (skinny black bar labelled ' θ_c ' in the main text Figure 2). The sensitivity of the ¹⁷O acquisition was improved by echo train acquisition using CPMG implementing central transition selective refocusing pulses. Twenty echoes, truncated to maximize sensitivity at the expense of resolution⁴ were acquired with an echo shift of one rotor period. For experiments at hole burning periods of 2, 4, 8 and $16\tau_{\rm r}$, the following respective number of scans were collected:

<u>20% enriched C3A</u>: 4, 8, 16, 40 <u>3% enriched C3A</u>: 16, 32, 64, 512 <u>5% enriched C3S</u>: 256, 512, 768, 1536

Reconstruction of CPMG data involved a matching procedure which exploited the sampling synchronicity of the windowed acquisition. The echo train was cut and appended as a function of echo count k. The amplitude of each data point comprising the kth echo by the filter function

$$L(\tau_k) = \exp(-(\tau_k/T)^{\beta})$$
,

where τ_k is the time after excitation for the k^{th} echo top and the parameters T = 5 ms and $\beta = 0.77$ match the decay of the CPMG envelope. The $L(\tau_k)$ apodized echo dimension is summed out leaving the signal of an echo with a significant sensitivity enhancement. Processing of the reconstructed signal continued with partial integration of the ¹⁷O line shape over the most intense points. No zero filling was used in this process. The resulting intensities were left as a function of normalized to noise. All spectral processing was carried out using the macOS application RMN, versions 1.8.4 or 1.8.6.⁵

III. Analysis of high-resolution ¹⁷O spectra

The high-resolution ¹⁷O spectra of the hydrated tricalcium aluminate samples are shown in Figure S2. These spectra are typical of the MAS NMR powder pattern of a single ¹⁷O site broadened by the second-order quadrupolar interaction. The spectra exhibit well-defined shoulders and sharp singularities, confirming the hydrated tricalcium aluminate phase corresponds to crystalline katoite, $Ca_3Al_2(OH)_{12}$.

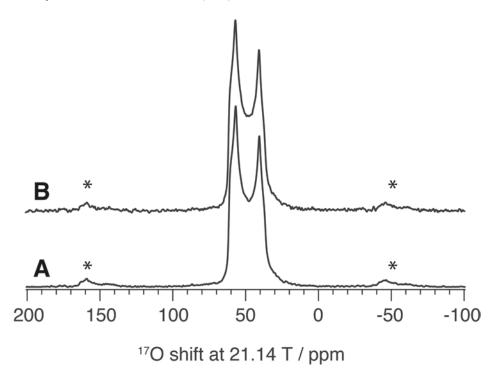


Figure S2. High-resolution ¹⁷O spectra at 21.14 T, 95 K, and 12.5 kHz MAS of (A) Ca₃Al₂O₆ (20%) with 128 scans, 5 s recycle delay, (B) Ca₃Al₂O₆ (3%) with 1024 scans, 2 s recycle delay. Proton decoupling with SPINAL64 and an rf amplitude of 125 kHz was applied during acquisition. Approximate frequency referencing is given with respect to the ¹H spectrum of the Ca₃Al₂O₆ (20%) sample assuming its maximum is at 4 ppm (split between water and hydroxyl type signals). Asterisks indicate spinning sidebands.

The hydration of tricalcium silicate is more complicated than tricalcium aluminate since calcium hydroxide is formed as a coexisting phase using our method of hydration. The high-resolution ¹⁷O spectra of the hydrated tricalcium silicate sample is shown in Figure S3. This spectrum is dominated by broad features which were assigned to different C-S-H oxygen environments by Kong and Kirkpatrick:⁶ between 100 and 130 ppm (silicate nonbridging oxygens), 30 ppm and 80 ppm (Ca-OH moieties and silicate bridging oxygens), and a low intensity tail going down to about -40 ppm (silanols and bound water). In addition to these broad features, the appearance of modest singularities at 44 ppm and 62 ppm correspond to ¹⁷O in the secondary phase, crystalline Ca(OH)₂. We quantify the fraction of ¹⁷O in each phase by modeling the shift distribution of the environments of ¹⁷O in C-S-H as three separate normal distributions, roughly corresponding to the regions described above. Fitting to a normal distribution is expected to be an adequate approximation for the C-S-H environments because the high magnetic field at which the measurements were performed reduces the skewedness of the line shape induced by the second-

order quadrupolar interaction while at the same time amplifies the Gaussian broadening due to chemical shift disorder. The shift distribution of ¹⁷O in Ca(OH)₂ is modeled as MAS NMR powder pattern of a single ¹⁷O site broadened by the second-order quadrupolar interaction. This powder pattern was calculated using $C_q = 7.0$ MHz, $\eta_q = 0$, and an isotropic chemical shift of $\delta_{iso} = 74$ ppm, in line with previously reported values.⁶ This fixes the shape and frequency distribution of the Ca(OH)₂ signal, with only the amplitude allowed to vary. The shift, width, and amplitude of the normal distributions were allowed to vary freely. The result of the fit is shown as the decomposition shown in Figure S3. We see that Ca(OH)₂ is a minor constituent, with 18% of the total ¹⁷O present in this phase, leaving 82% present in the C-S-H. The residuals shown in Figure S3 appear as noise, improving the confidence in our parameterization.

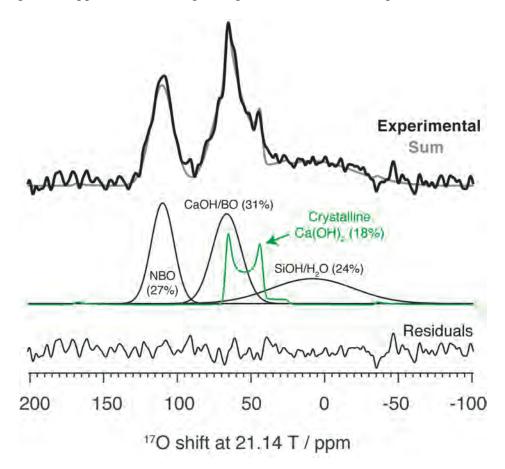


Figure S3. Quantitative decomposition of the high-resolution ¹⁷O direct excitation MAS NMR spectrum of Ca₃SiO₅ (5%) (top, black) into three Gaussian components (center, black) and a second-order quadrupole powder pattern (center, green). The former represents the ¹⁷O environments in the poorly ordered calcium silicate hydrate phase whereas the latter corresponds to the spectrum of a coexisting crystalline Ca(OH)₂ phase. The residuals (bottom) are the difference between the experimental spectrum and the sum of the calculated components (top, gray). The contribution of each component to the total area of the calculated spectrum is given in percent next to the corresponding oxygen species (NBO – silicate non-bridging oxygen; BO – silicate bridging oxygen). Spectrum was acquired with 512 scans, 10 s recycle delay at 21.14 T, 95 K, and 12.5 kHz MAS. Proton decoupling with SPINAL64 and an rf amplitude of 125 kHz was applied during acquisition. Approximate frequency referencing is given with respect to the ¹H spectrum of the Ca₃Al₂O₆ (20%) sample by assuming its maximum is at 5 ppm.

IV. Data reproducibility

Owing to the high rf duty cycle of the hole burning pulse sequence (main text Figure 2), we occasionally observed bizarre behavior from circuit detuning in which the signal response of a repeating hole burning experiment would become erratic and/or diminish when the hole burning period exceeded $8\tau_R$. This is illustrated in Figure S4 for repeated experiments at the shortest BR24 cycle time we could achieve that was commensurate with the 330 µs rotor period: $\tau_c \approx 66$ µs (giving $\tau_c/\tau_R \approx 5$).

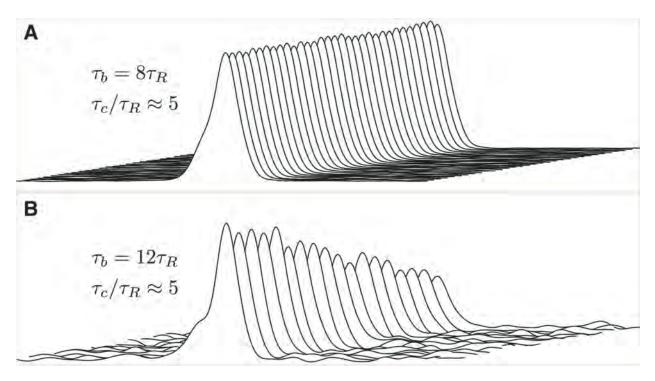


Figure S4. Reconstruction of (truncated) ¹⁷O CPMG intensities for repeated hole burning experiments to test reproducibility. Horizontal axis is frequency and the span of each spectrum is constant. The first iteration of the experiment begins with the spectrum on the left. Sequentially to the right, each spectrum is the result of an identical hole burning experiment executed immediately following the preceding iteration. (A) Intensity profile of hole burning experiments at $\tau_b = 8\tau_R$ (2.64 ms) and $\tau_c \approx 66 \,\mu$ s. The intensities of subsequent iterations are constant to within noise and the experiment is reproducible. (B) Intensity profile of hole burning experiments at a slightly longer $\tau_b = 12\tau_R$ (3.96 ms) and the same $\tau_c \approx 66 \,\mu$ s. The intensities of subsequent iterations diminish and reproducibility is lost.

For $\tau_b > 8\tau_R$ and $\tau_c \approx 66 \ \mu s$ it would often take minutes for the circuit to recover. Reproducibility was greatly improved by using a longer BR24 cycle time $\tau_c \approx 82 \ \mu s \ (\tau_c/\tau_R \approx 4)$ in which the decoupling pulse lengths and powers were unchanged but longer delays were incorporated into each decoupling period, thus reducing the duty cycle.

The experiment we analyze for diffusion coefficients use the longer $\tau_c \approx 82 \ \mu s$. No drop in decoupling efficiency relative to $\tau_c \approx 66 \ \mu s$ was apparent.

V. Data analysis

Data fitting was carried out using scripts written for *gnuplot*. The fit to the main text Eq. (5) was carried out in a multibranch fashion, fitting branches for each sample and at hole burning periods of 2, 4, 8 and $16\tau_r$ simultaneously. Specific constraints were as follows:

<u>Constraints across entire fit</u>: $T_1 = 11.713127$ ms (determined from prior fit of main text Eq. (5) to the Ca₃Al₂O₆ (20%) branch) $M_1 = 0$ $M_2 = 1$ (for 5% Ca₃SiO₅ only, due to poorer SNR resulting from greater ¹⁷O dilution) $R (2\tau_R) = 0.1722$ nm (constant used to return D_H from the characteristic diffusion time) $R (4\tau_R) = 0.2170$ nm $R (8\tau_R) = 0.2734$ nm $R (16\tau_R) = 0.3444$ nm

Fit but constrained to be equal across sample branches: Advancement parameters Δt (property of hole burning; should be sample independent)

<u>Free parameters for each sample and τ_{b} :</u> M_2 (except for Ca₃SiO₅ (5%) as noted above) $D_{\rm H}$

The results are given in the following table. We suggest that the fit errors reported in the table, insofar as they reflect upon uncertainty in the extrapolated "macroscopic" values of $D_{\rm H}$, are unimportant in comparison to the uncertainty in the selection of R.

Parameter	Sample	Hole burning interval			
		$2\tau_R$	$4\tau_R$	$8\tau_R$	$16\tau_R$
	Ca ₃ Al ₂ O ₆ (20%)	156 ± 11	204 ± 15	222 ± 12	268 ± 24
$D_{ m H} / { m nm^2 s^{-1}}$	Ca ₃ Al ₂ O ₆ (3%)	165 ± 11	200 ± 13	214 ± 11	264 ± 23
	Ca ₃ SiO ₅ (5%)	71 ± 7	105 ± 12	120 ± 12	187 ± 30
M ₂ / %	Ca ₃ Al ₂ O ₆ (20%)	88.4 ± 1.1	81.5 ± 1.3	73.2 ± 0.9	65.4 ± 1.5
	Ca ₃ Al ₂ O ₆ (3%)	100.4 ± 1.1	94.5 ± 1.1	90.9 ± 1.0	76.1 ± 1.6
$\Delta t / \mu s$		2.2 ± 1.6	13.5 ± 1.9	22.1 ± 2.1	24.0 ± 4.4

Table S1. Complete table of parameters determined by fitting the hole burning data to Eq. (5) of the main text, subject to the constraints described in this section. Error ranges corresponds to asymptotic standard errors reported by the goodness of fit routine.

Plots in the manner of the main text Figure 3 for all samples, and which includes the data for the hole burning interval $\tau_b = 5.28 \text{ ms} (16\tau_R)$, are plotted below as Figure S5.

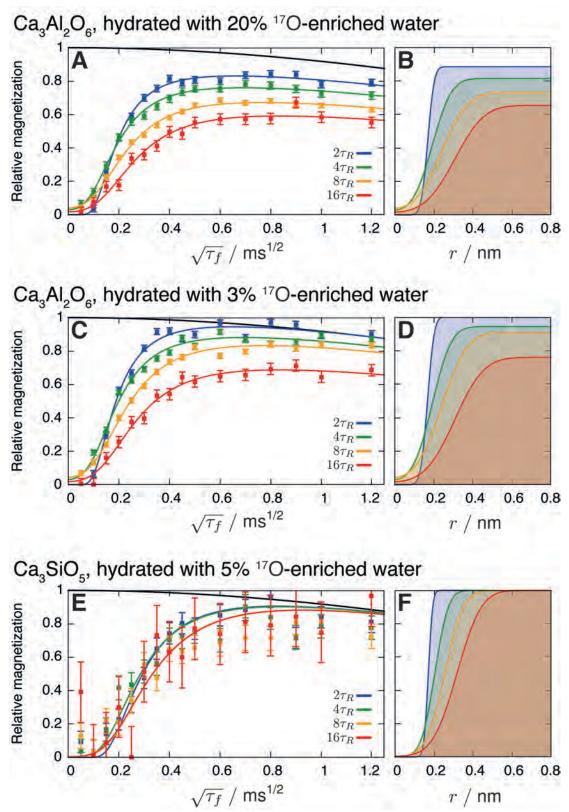


Figure S5. Temporal profiles of experiment hole filling recovery intensities, normalized to an experiment without REDOR pulses, for (A) $Ca_3Al_2O_6$ (20%), (B) $Ca_3Al_2O_6$ (3%), (C) Ca_3SiO_5 (5%). Initial radial magnetization profiles in the model context are given in panels B, D, and F, respectively for each sample.

For reasons stated in section III above, we excluded the $16\tau_r$ data from our primary discussion. This data (at the lower duty cycle $\tau_c/\tau_R \approx 4$) does not seem internally inconsistent, however, and should perhaps be included. We find that our results are not significantly affected if we include the $16\tau_r$ data in the extrapolation to infinite hole burning period, as shown in Figure S6 and Table S2. The consistently and anomalously high D_H we analyze in the $16\tau_r$ data, if legitimate, could indicate a transition to a regime where diffusion adopts a different character, perhaps due to the diminishing importance of nondiffusive spin dynamics or diffusion anisotropy at the larger hole sizes.

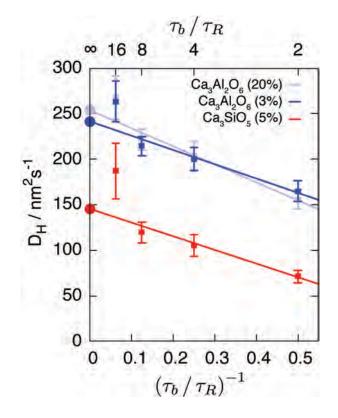


Figure S6. Determination of the macroscopic proton spin diffusion coefficients D_H^{∞} by extrapolation from finite hole burning intervals, including the parameters determined by analysis of the 16 τ_r data. The data points and error bars correspond to the parameters given in Table S1.

Sample	D_H^{∞} / nm ² s ⁻¹	D_H^∞ / nm ² s ⁻¹	
	Excluding $16\tau_R$	Including $16\tau_R$	
Ca ₃ Al ₂ O ₆ (20%)	245 ± 4	254 ± 12	
Ca ₃ Al ₂ O ₆ (3%)	232 ± 2	242 ± 14	
Ca ₃ SiO ₅ (5%)	137 ± 2	145 ± 16	

Table S2. Comparison of the macroscopic proton spin diffusion coefficients, D_H^{∞} , between analyses which include and exclude the $16\tau_r$ data.

VI. Hole overlap

In our model the M_2 coefficient can be physically interpreted in terms of hole overlap and the concentration of ¹⁷O nuclei in the samples, according to

$$M_2 = (1 - c_S V_{\text{hole}}) + M_{\Delta}$$

Here, c_S is the number concentration of ¹⁷O spins in the sample, V_{hole} is the effective volume of a polarization hole due to dephasing, and M_{Δ} is defined as the excess fraction of unburned polarization. The quantity $(1 - c_S V_{hole})$ is the residual magnetization assuming all holes burn to the maximum extent of their capability, which is the case when holes do not overlap. We therefore expect $M_{\Delta} > 0$ as shared hole volume permits an excess of unburned magnetization. In the limit of infinite ¹⁷O dilution, and as hole volume shrinks, we expect $M_{\Delta} \rightarrow 0$.

The parameter c_s can be calculated assuming the bulk density of katoite, 2.76 g/cm³, the hydrated stoichiometry Ca₃Al₂O₆(H₂O)₆, and completeness of oxygen exchange between water and tricalcium aluminate. Using the information in section I, these assumptions lead to actual ¹⁷O enrichment of 14.9% and 2.2% along with c_s values of 7.87 nuclei/nm³ and 1.18 nuclei/nm³ for the nominally Ca₃Al₂O₆ (20%) and Ca₃Al₂O₆ (3%) samples, respectively. V_{hole} is calculated assuming the effective radius of the spherical hole is determined by $r_{0.5}$.

In Fig S7A, experimentally derived values of the M_2 coefficient are plotted against $r_{0.5}$. The difference between the solid lines and the experimental data points is M_{Δ} . For the Ca₃Al₂O₆ (3%) sample, we see that M_2 coefficients cluster around the solid line, suggesting that hole overlap is not significant at this ¹⁷O concentration. For the Ca₃Al₂O₆ (20%) sample, however, we observe substantially nonzero values of M_{Δ} , especially at $\tau_b = 8\tau_R$, when the hole volume is 0.086 nm³. This is consistent with our expectation that higher ¹⁷O concentrations and larger hole volumes lead to more significant hole overlap.

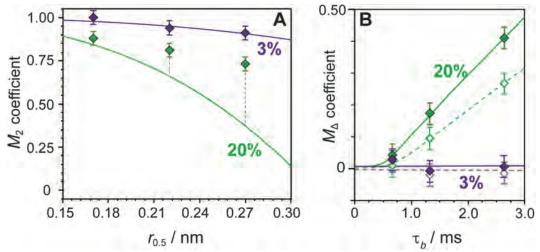


Figure S7. (A) Values of the M_2 coefficient versus hole-size $(r_{0.5})$ for the 3%-enriched (purple) and 20%-enriched (green) hydrated Ca₃Al₂O₆. The solid lines are graphs of $(1 - c_s V_{hole})$, presenting a lower bound on M_2 . Dashed vertical lines correspond to M_{Δ} . (B) Excess fraction of unburned polarization, M_{Δ} , for the two samples, plotted against the duration of hole burning. Filled and unfilled diamonds correspond to using $r_{0.5}$ and r_{nat} , respectively, to approximate the size of the hole. The solid ($r_{0.5}$) and dashed (r_{nat}) lines serve only to guide the eye.

The excess fraction of unburned polarization we calculate also depends on the accuracy of the effective hole volume, V_{hole} . In Fig. S7B, we plot M_{Δ} for two sets of estimates for V_{hole} , one based upon $r_{0.5}$ and the other r_{nat} . The smaller hole-size, r_{nat} , leads to smaller predicted values of M_{Δ} . These results are summarized in Table S3.

Sample A	Actual ¹⁷ O concentration	$R = r_{0.5}$		$R = r_{\rm nat}$	
		$V_{\rm hole}$ / nm ³	M_{Δ} / %	$V_{\rm hole}$ / nm ³	M_Δ / %
Ca ₃ Al ₂ O ₆ (20%)	14.9%	0.0214	5.2	0.0171	1.9
		0.0428	15.2	0.0342	8.4
		0.0856	40.6	0.0685	27.1
Ca ₃ Al ₂ O ₆ (3%)	2.24%	0.0214	2.9	0.0171	2.4
		0.0428	-0.4	0.0342	-1.5
		0.0856	1.0	0.0685	-1.0

Table S3. Numerical values of M_{Δ} for the hydrated tricalcium aluminate samples, calculated for two sets of effective hole radii and three different hole burning intervals. Estimated 2σ uncertainties for the M_{Δ} are $\pm 3\%$.

VII. General solution of the radial diffusion equation for the spherical well

We are solving the three-dimensional isotropic diffusion equation, Eq. (3) of the main text

$$\frac{\partial M}{\partial t} = D\nabla^2 M,$$

with $M \equiv M(r, \theta, \phi, t)$, and the diffusion coefficient D, for the initial value problem

$$M(r,0) = \begin{cases} M_1 \text{ if } r < R, \\ M_2 \text{ if } r > R, \end{cases}$$
$$\frac{\partial M}{\partial r}\Big|_{r=0} = 0, \forall t,$$
$$\lim_{r \to \infty} M(r,t) = M_2, \forall t,$$

and *M* is independent of θ and ϕ . Eq. (3) can be transformed into the homogenous Helmholtz equation,

$$(\nabla^2 + k^2)g = 0$$

where $g \equiv g(r, \theta, \phi)$, is stripped of its time-dependence by its relation to the "normal modes"

$$M_k(r,\theta,\phi,t) = g_k(r,\theta,\phi)e^{-Dk^2t}$$

indexed by the eigenvalue k, which has units of inverse length.

The solutions to the homogeneous Helmholtz equation for cases of spherical symmetry are given by the spherical wave solutions $g_k(r) = e^{-ikr}/r$. From this we have the general solution

$$M(r,t) = \int_{-\infty}^{+\infty} A(k) \frac{e^{-ikr}}{r} e^{-Dk^2t} dk.$$

The combination of modes which solves the initial value problem is given by the function A(k). There is no need to solve for A(k) in terms of k, as the above equation can be transformed into

$$M(r,t) = M_2 + \frac{M_1 - M_2}{\sqrt{4\pi Dt}} \frac{1}{r} \int_{-R}^{+R} r' e^{-\frac{(r-r')^2}{4Dt}} dr'$$

by use of the convolution theorem of Fourier transforms and application of the boundary conditions. The integral in this expression can be evaluated to yield the analytic solution

$$M(r,t) = M_2 + \frac{M_1 - M_2}{2} \left[\left(erf\left(\frac{r+R}{\sqrt{4Dt}}\right) - erf\left(\frac{r-R}{\sqrt{4Dt}}\right) \right) + \sqrt{\frac{4Dt}{\pi}} \frac{1}{r} \left(e^{-\frac{(r+R)^2}{4Dt}} - e^{-\frac{(r-R)^2}{4Dt}} \right) \right]$$

from which, upon taking the limit as $r \rightarrow 0$, we obtain Eq. (4) in the main text.

VIII. Additional references

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